PhD Course
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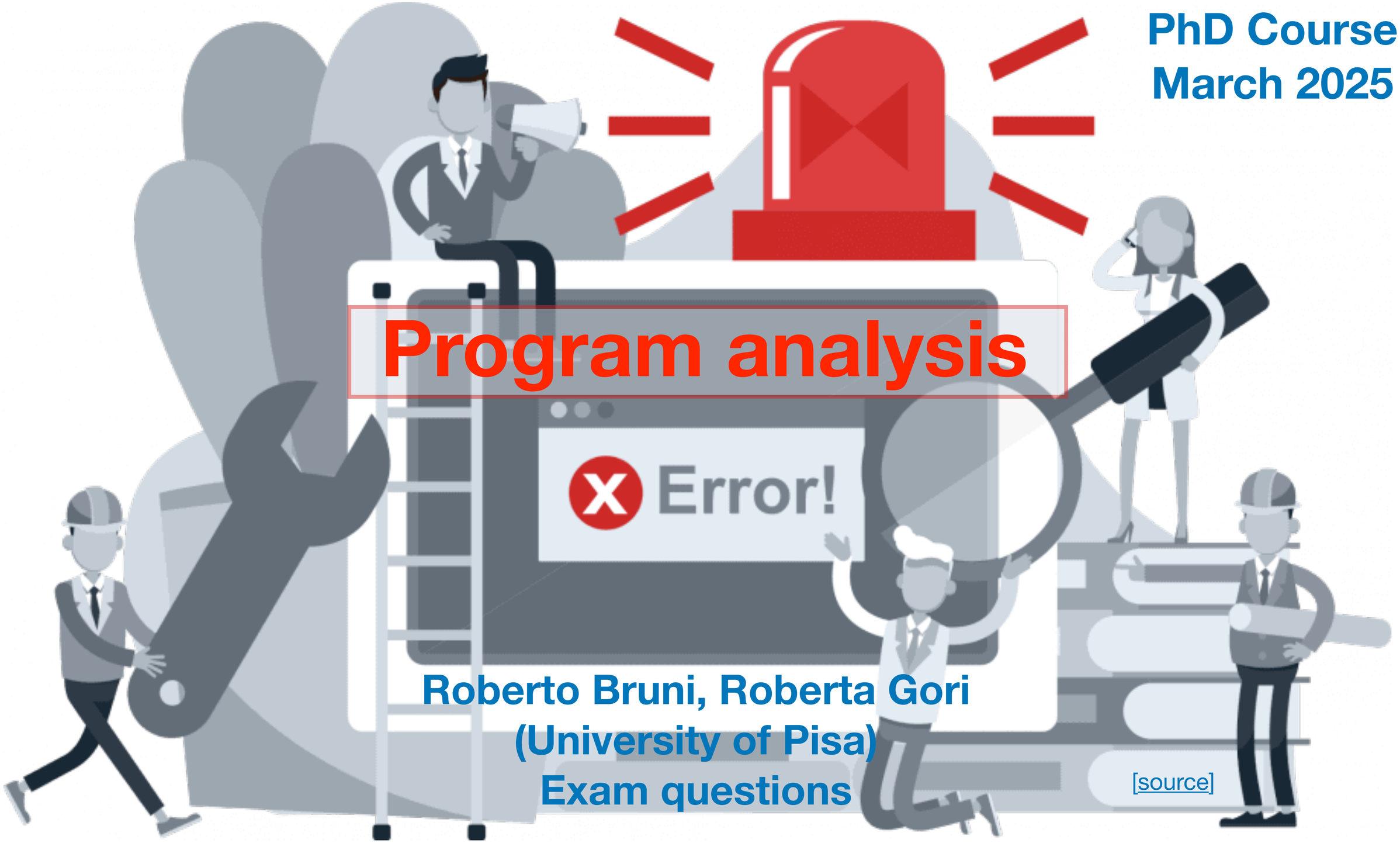
Program analysis



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Exam questions

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Exam questions

Exam question #1

Complete the proof of total correctnes

lexicographic order

$\{\text{true}\}$

$k := 1;$

$r := x;$

$\{k \geq 0 \wedge f^k(r) = f(x)\}$ $\quad t = (|101 - r + 10k|, k)$

while $k > 0$ do

  if $r > 100$ then

    $r := r - 10;$

    $k := k - 1$

  else

    $r := r + 11;$

    $k := k + 1$

$\{k = 0 \wedge f^k(r) = f(x)\} \Rightarrow \{r = f(x)\}$

$$f(x) \triangleq \begin{cases} 91 & x \leq 100 \\ x - 10 & \text{otherwise} \end{cases}$$

$$f(x) \triangleq \begin{cases} f(f(x + 11)) & x \leq 100 \\ x - 10 & \text{otherwise} \end{cases}$$

Exam question #2

Complete the proof of correspondence $\{P\}r\{Q\} \iff (\neg P)r(\neg Q)$ between HL and NC triples

One implication, the other is similar

$[\![r]\!]P \subseteq Q \implies [\![\overleftarrow{r}]\!]\neg Q \subseteq \neg P$

$\sigma' \in \neg Q \implies \sigma' \notin Q$

Hence, $\sigma' \notin [\![r]\!]P \iff \forall \sigma \in P.\sigma' \notin [\![r]\!]\sigma$

Since $\sigma \in [\![\overleftarrow{r}]\!]\sigma' \Leftrightarrow \sigma' \in [\![r]\!]\sigma$ $\iff \forall \sigma \in P.\sigma \notin [\![\overleftarrow{r}]\!]\sigma'$

$\iff P \cap [\![\overleftarrow{r}]\!]\sigma' = \emptyset$

$\iff [\![\overleftarrow{r}]\!]\sigma' \subseteq \neg P$

Since it holds for all $\sigma' \in Q$ $\iff [\![\overleftarrow{r}]\!]\neg Q \subseteq \neg P$

Exam question #3

Consider the imprecise list segment definition below

$$\mathsf{ils}(a_1, a_2) \triangleq (a_1 = a_2 \wedge \mathsf{emp}) \vee (\exists v \,.\, a_1 \mapsto v * \mathsf{ls}(v, a_2))$$

Prove that $\mathsf{ils}(a_1, a_2) \not\equiv \mathsf{ls}(a_1, a_2)$ by finding a state that distinguishes $\mathsf{ls}(11,11)$ from $\mathsf{ils}(11,11)$

$$\mathsf{ls}(a_1, a_2) \triangleq (a_1 = a_2 \wedge \mathsf{emp}) \vee (a_1 \neq a_2 \wedge \exists v \,.\, a_1 \mapsto v * \mathsf{ls}(v, a_2))$$

Exam question #4

Prove the SepSIL triple $\langle\langle p \mapsto \text{nil} * \text{true} \rangle\rangle\ c\ \langle\langle i = 0 \rangle\rangle$ where

$$c \triangleq i := 0\,;\, q := *\,p\,;\, \text{while } (q \neq \text{nil}) \text{ do } \{\ q := *\,q\,;\, i := i + 1\ \}$$

$$i := 0\,;$$
$$q := [p]\,;$$
$$(\ \ (q \neq \text{nil}?)\,;\, q := [q]\,;\, i := i + 1\ )^{\star};$$
$$(q = \text{nil}?)$$

Exam question #5

Consider the abstract domain Sign’ in the figure

1. Define the corresponding α and γ.

2. Does it admit a complete abstract multiplication?